

SM3 1.3: Polynomial Long Division Key

Vocabulary: Polynomial, Descending Order, Dividend, Divisor, Quotient, Remainder

Vocabulary Problems: Answer the questions about each function:

1) $a(x) = \frac{4x^2 + 2x + 3}{x - 8}$

The dividend of $a(x)$ is: $4x^2 + 2x + 3$

The divisor of $a(x)$ is: $x - 8$

2)

$$\begin{array}{r} x + 4 \\ x - 1 \overline{) x^2 + 3x + 7} \\ \underline{-(x^2 - x)} \\ 4x + 7 \\ \underline{-(4x - 4)} \\ 11 \end{array}$$

The dividend is: $x^2 + 3x + 7$

The divisor is: $x - 1$

The remainder is: 11

The $\frac{\text{remainder}}{\text{divisor}}$ is: $\frac{11}{x-1}$

The quotient is: $x + 4 + \frac{11}{x-1}$

Setting up: Set up the long divisions with appropriate placeholders, but do not find the quotients:

3)
$$\frac{75x^4 - 3}{x^2 - 4x - 8}$$

$$x^2 - 4x - 8 \overline{) 75x^4 + 0x^3 + 0x^2 + 0x - 3}$$

4)
$$\frac{2 - 5x + x^3}{1 - 3x^2}$$

$$-3x^2 + 0x + 1 \overline{) x^3 + 0x^2 - 5x + 2}$$

Problems: Find each quotient.

5)
$$\frac{x^2 + 10x + 24}{x + 6}$$

$$\begin{array}{r} x + 4 \\ x + 6 \overline{) x^2 + 10x + 24} \\ \underline{-(x^2 + 6x)} \\ 4x + 24 \\ \underline{-(4x + 24)} \\ 0 \end{array}$$

6)
$$\frac{x^2 - 25}{x - 5}$$

$$\begin{array}{r} x + 5 \\ x - 5 \overline{) x^2 + 0x - 25} \\ \underline{-(x^2 - 5x)} \\ 5x - 25 \\ \underline{-(5x - 25)} \\ 0 \end{array}$$

7)
$$\frac{5x^2 + 8x + 7}{x + 4}$$

$$\begin{array}{r} 5x - 12 + \frac{55}{x + 4} \\ x + 4 \overline{) 5x^2 + 8x + 7} \\ \underline{-(5x^2 + 20x)} \\ -12x + 7 \\ \underline{-(-12x - 48)} \\ 55 \end{array}$$

$$8) \frac{x^3 + 4x^2 + 9x + 36}{x + 4}$$

$$\begin{array}{r} x + 4 \overline{) x^3 + 4x^2 + 9x + 36} \\ \underline{-(x^3 + 4x^2)} \\ 0 + 9x + 36 \\ \underline{-(9x + 36)} \\ 0 \end{array}$$

$$9) \frac{2x^3 - 2x^2 - x - 3}{x - 1}$$

$$\begin{array}{r} x - 1 \overline{) 2x^3 - 2x^2 - x - 3} \\ \underline{-(2x^3 - 2x^2)} \\ 0 - x - 3 \\ \underline{-(-x + 1)} \\ -4 \end{array}$$

$$10) \frac{x^3 + 5x}{x^2 - 2}$$

$$\begin{array}{r} x^2 + 0x - 2 \overline{) x^3 + 0x^2 + 5x + 0} \\ \underline{-(x^3 + 0x^2 - 2x)} \\ 7x + 0 \end{array}$$

$$11) (x^2 + 5x - 7) \div (x - 2)$$

$$\begin{array}{r} x - 2 \overline{) x^2 + 5x - 7} \\ \underline{-(x^2 - 2x)} \\ 7x - 7 \\ \underline{-(7x - 14)} \\ 7 \end{array}$$

$$12) (x^2 + 7)(x + 3)^{-1}$$

$$\begin{array}{r} x + 3 \overline{) x^2 + 0x + 7} \\ \underline{-(x^2 + 3x)} \\ -3x + 7 \\ \underline{-(-3x - 9)} \\ 16 \end{array}$$

$$13) (x^3 - x^2 + x - 2)(x^2 + 3x - 1)^{-1}$$

$$\begin{array}{r} x^2 + 3x - 1 \overline{) x^3 - x^2 + x - 2} \\ \underline{-(x^3 + 3x^2 - x)} \\ -4x^2 + 2x - 2 \\ \underline{-(-4x^2 - 12x + 4)} \\ 14x - 6 \end{array}$$

$$14) \frac{x^4 - 1}{x - 1}$$

$$\begin{array}{r} x - 1 \overline{) x^4 + 0x^3 + 0x^2 + 0x - 1} \\ \underline{-(x^4 - x^3)} \\ x^3 + 0x^2 \\ \underline{-(x^3 - x^2)} \\ x^2 + 0x \\ \underline{-(x^2 - x)} \\ x - 1 \\ \underline{-(x - 1)} \\ 0 \end{array}$$

$$15) \frac{2x - 5}{x^2 - 3x - 7}$$

$$x^2 - 3x - 7 \overline{) 2x - 5}$$

The denominator has more power than the numerator. When divided, the numerator is the remainder and the quotient is the original problem!

Composite Problems: Find each quotient.

$$16) \frac{(x^2 + 4) + (2x^2 + x - 5)}{x + 10}$$

$$\frac{3x^2 + x - 1}{x + 10}$$

$$\begin{array}{r}
 3x - 29 + \frac{289}{x + 10} \\
 x + 10 \overline{) 3x^2 + x - 1} \\
 \underline{-(3x^2 + 30x)} \\
 -29x - 1 \\
 \underline{-(-29x - 290)} \\
 289
 \end{array}$$

$$17) \frac{(2x - 1)^3}{x - 2}$$

$$\frac{8x^3 - 12x^2 + 6x - 1}{x - 2}$$

$$\begin{array}{r}
 8x^2 + 4x + 14 + \frac{27}{x - 2} \\
 x - 2 \overline{) 8x^3 - 12x^2 + 6x - 1} \\
 \underline{-(8x^3 - 16x^2)} \\
 4x^2 + 6x - 1 \\
 \underline{-(4x^2 - 8x)} \\
 14x - 1 \\
 \underline{-(14x - 28)} \\
 27
 \end{array}$$

$$18) \frac{(x - 3)(x^2 - x - 6)}{x + 2}$$

$$\frac{x^3 - 4x^2 - 3x + 18}{x + 2}$$

$$\begin{array}{r}
 x^2 - 6x + 9 \\
 x + 2 \overline{) x^3 - 4x^2 - 3x + 18} \\
 \underline{-(x^3 + 2x^2)} \\
 -6x^2 - 3x + 18 \\
 \underline{-(-6x^2 - 12x)} \\
 9x + 18 \\
 \underline{-(9x + 18)} \\
 0
 \end{array}$$

$$19) \frac{(x + 5)^3}{x - 1}$$

$$\frac{x^3 + 15x^2 + 75x + 125}{x - 1}$$

$$\begin{array}{r}
 x^2 + 16x + 91 + \frac{216}{x - 1} \\
 x - 1 \overline{) x^3 + 15x^2 + 75x + 125} \\
 \underline{-(x^3 - x^2)} \\
 16x^2 + 75x + 125 \\
 \underline{-(16x^2 - 16x)} \\
 91x + 125 \\
 \underline{-(91x - 91)} \\
 216
 \end{array}$$

Application: Use polynomial long division to find the quotient.

- 20) The volume of a classroom is given by $v(x) = x^3 - 9x^2 + 23x - 15 \text{ ft}^3$. The surface area of the floor of the classroom is given by $x^2 - 4x + 3 \text{ ft}^2$. Assuming the classroom is a rectangular solid, find an expression for the height, in ft of the classroom.

The area of a rectangle is given by: $A = b_{\text{rectangle}} \cdot h_{\text{rectangle}}$

The volume of a rectangular solid is given by: $V = b_{\text{rectangle}} \cdot h_{\text{rectangle}} \cdot h_{\text{solid}}$

By substitution, $V = A \cdot h_{\text{solid}}$

We divide to solve for h_{solid} : $\frac{V}{A} = h_{\text{solid}}$

$$\begin{array}{r} x^2 - 4x + 3 \overline{) x^3 - 9x^2 + 23x - 15} \\ \underline{-(x^3 - 4x^2 + 3x)} \\ -5x^2 + 20x - 15 \\ \underline{-(-5x^2 + 20x - 15)} \\ 0 \end{array}$$